AXISYMMETRIC FLOW FAR FROM A BODY IN THE VICINITY OF THE AXIS WITH MACH NUMBER M_∞ CLOSE TO UNITY

(OSESIMMETRICHNOE TECHENIE VDALI OT TELA V OKRESTNOSTI OSI PRI CHISLE M_∞, BLIZKOM K EDINITSE)

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F.V. SHUGAEV
(Moscow)

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Suppose that an axisymmetric blunt body is placed in a stream, the Mach number M_∞ of which differs only slightly from unity. Let us consider the flow in the neighborhood of the axis at a great distance from the body upstream. The origin of coordinates is located at the critical point of the body, the x-axis coincides with the direction of the undisturbed stream. In the neighborhood of the axis

$$u = f(\tau_1) + \sum_1^\infty \alpha_i(\tau_1) y^{2i}, \qquad v = \sum_1^\infty \beta_i(\tau_1) y^{2i-1}, \qquad \tau = \frac{2\Psi(x, y)}{y^2}$$

Here u and v are the components of velocity along the axes x and y expressed in terms of the critical velocity, $\Psi(x,y)$ is the stream function, $f(\tau_1)$ is the value of the velocity on the axis [1].

It is convenient to introduce a change of variables $\tau = \varepsilon\tau_1$, where

$$\varepsilon = \begin{cases} k^{-1/2}\left[\gamma\left(1 - \dfrac{1}{ak^2}\right)\right]^{1/(\varkappa-1)}, & M_\infty > 1, \qquad a = 1 + \dfrac{2}{\varkappa - 1}\dfrac{1}{M_\infty^2}, \qquad \gamma = \dfrac{\varkappa + 1}{2} \\ k^{1/2}\left[\gamma\dfrac{(a-1)}{a}\right]^{1/(\varkappa-1)}, & M_\infty < 1, \qquad k = \dfrac{\varkappa + 1}{\varkappa - 1}\dfrac{1}{a}, \qquad \varkappa = \dfrac{c_p}{c_v} \end{cases}$$

The straight line $\tau = \varepsilon$ is [1] the shock wave $(M_\infty > 1)$ or the undisturbed stream at infinity $(M_\infty \leqslant 1)$. The straight line $\tau = 1$ is a limit line.

By the same token, the limit line cuts the axis [2] at the point $\tau = 1$. From condition $D(\tau, y) / D(u, v) = 0$, defining the existence of a limit line, we find

$$F = A_1B_2 - A_2B_1 \tag{1}$$

where A_1, A_2, B_1 and B_2 are the coefficients of the derivatives of u and v with respect to τ_1 in the equations of gas dynamics in the plane $\tau_1 y$. If $\tau = L(y)$ is the equation of the limit line, then

$$\frac{dL}{dy} = -\left[\frac{\partial F/\partial y}{\partial F/\partial \tau}\right]_{\tau=L}$$

As a result of the boundedness of the numerator and the tendency to infinity of the denominator L = const = 1 . The coefficients α_1 and β_1 are represented thus: $\alpha_1 = \alpha_{10} + \alpha_{11}$, $\beta_1 = \beta_{10} + \beta_{11}$, where α_{10}, β_{20} respectively, are the values of α_1 and β_1 when $M_\infty = 1$, whilst α_{11} and β_{11} are the corrections required by the deviation of M_∞ from unity. The quantities α_{10} and β_{10} are determined by the expression of the perturbing potential [3] for transonic flow ($M_\infty = 1$)

$$\Phi = y^{-2/7} g(C\zeta) C^{-3}, \qquad \zeta = (\varkappa + 1)^{-1/3} xy^{-4/7} \tag{2}$$

where C is an arbitrary constant. Making use of Formula (2), we find that

$$\alpha_{10} = 6c^2 \Delta\tau^{5/3}, \quad \beta_{10} = 3c\Delta\tau^{4/3}, \quad \Delta\tau = 1 - \tau, \quad c = C^{7/3} (\varkappa + 1)^{-4/9} \tag{3}$$

The equation of motion in the variables τy can be written in the form [1]

$$\frac{\partial}{\partial \tau} (\varepsilon k^{-1/2} p + 2\tau u) y = \frac{\partial}{\partial y} (y^2 u)$$

where p is the pressure, expressed in terms of the dynamic head $\frac{1}{2}\rho_\infty u_\infty^2$, u_∞ and ρ_∞ are the velocity and density of the stream at infinity. Integrating this equation with respect to τ from $1 - \Delta\tau$ to 1 and restricting ourselves to terms of the lowest degree with respect to y , we have

$$\frac{4}{\varkappa + 1} \Delta M_\infty^2 h\alpha_1(\varepsilon) [1 - (1 - \Delta\tau) N] + \frac{\beta_1^2}{2} N - \frac{\beta_1^2(1)}{2} = 2 \int\limits_{1-\Delta\tau}^{1} \alpha_1 d\tau \tag{4}$$

$$h = 1 + O(\Delta M_\infty), \qquad \Delta M_\infty = |M_\infty - 1|, \qquad N = \gamma^{1/(\varkappa-1)} \left(1 - \frac{\varkappa - 1}{\varkappa + 1} f^2\right)^{1/(\varkappa-1)}$$

The function $f(\tau)$ is defined by the relation [1]

$$fN = \tau \qquad (f = 1 - \gamma^{-1/2} \Delta\tau^{1/2} + \ldots, (\tau \sim 1)) \tag{5}$$

The quantities $\alpha_1(\varepsilon)$ and $\beta_1(\varepsilon)$ with $M_\infty > 1$ must satisfy the relation on the shock wave

$$\beta_1^2(\varepsilon) = 4q\gamma^{-1}\alpha_1(\varepsilon) \Delta M_\infty^2, \qquad q = 1 + O(\Delta M_\infty), \qquad M_\infty > 1 \tag{6}$$

whilst, when $M_\infty < 1$, they must vanish (the condition that the stream is undisturbed at infinity)

$$\alpha_1(\varepsilon) = 0, \qquad \beta_1(\varepsilon) = 0, \qquad M_\infty < 1 \tag{7}$$

Expanding $\alpha_{11}(\tau)$ and $\beta_{11}(\tau)$ in series in the neighborhood of the point $\tau = 1$, and making use of equations (3) to (5) as well as conditions (1), (6) and (7) (retaining terms of the lowest degree in ΔM_∞ and assuming $\eta = \gamma^{-4/3}$), we obtain (8)

$$\alpha_1 = 6c^2 \Delta\tau^{1/3} [\Delta\tau^{4/3} - \eta \Delta M_\infty^{8/3}] + \ldots, \quad \beta_1 = 3c [\Delta\tau^{4/3} - \eta \Delta M_\infty^{8/3}] + \ldots \quad (M_\infty < 1)$$

$$\alpha_1 = 6c^2 \Delta\tau^{1/3} [\Delta\tau^{4/3} + {}^5/_3 \eta \Delta M_\infty^{8/3}] + \ldots, \quad \beta_1 = 3c [\Delta\tau^{4/3} + {}^5/_3 \eta \Delta M_\infty^{8/3}] + \ldots \quad (M_\infty > 1)$$

Accordingly, the corrections to the velocity components u and v at the shock wave ($\tau = \varepsilon$ and $M_\infty > 1$) expressed in terms of the critical velocity of the flow, are of different orders, namely

$$u_1 \sim \alpha_{11} y^2 \sim c_1 \Delta M_\infty^{10/3} y^2, \qquad v_1 \sim \beta_{11} y \sim c_2 \Delta M_\infty^{8/3} y \qquad (c_1, c_2 \neq 0)$$

Guderley [3] wrote down the potential for axisymmetric transonic flow ($M_\infty > 1$) in the form

$$\Phi_1 = a_0 y^{-2/7} g(C\zeta) + a_1 (M_\infty - 1)^{5/3} y^{8/7} g(\zeta, {}^8/_7) + \ldots \qquad (a_0, a_1 = \text{const}) \tag{9}$$

The correction, arising from the difference between M_∞ and unity, is proportional to $\Delta M_\infty^{5/3}$. This formula does not satisfy the boundary condition at the shock wave. Comparison with the results of the present paper shows the inapplicability of Formula (9) for flow at a great distance from the blunt body in the neighborhood of the axis.

From Equation (8) we can, in particular, obtain an asymptotic formula for the function $D(M_\infty)$ when $M_\infty \sim 1$ (D is the distance between the shock wave and the axisymmetric blunt body). The quantity D is equal [4] to

$$D = \frac{1}{2\varepsilon k^{1/2}} \int_0^\varepsilon \frac{p_1}{[\rho \partial v / \partial y]_{y=0}}$$

where ρ is the density relative to ρ_∞. Using (8) we find that when $M_\infty \sim 1$

$$D \sim \frac{1}{2} \int_0^\varepsilon \frac{d\tau}{[\rho \partial v / \partial y]_{y=0}} \sim \frac{1}{6} c^{-1} (\varkappa + 1)^{4/5} \int \frac{d\tau}{\Delta\tau^{4/3} \mp \frac{5}{3}\eta \Delta M^{8/3}_\infty}$$

Hence it follows that

$$D \sim 0.32353 (\varkappa + 1)^{11/5} C^{-7/5} (M_\infty - 1)^{-3/5}$$

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